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A Note on the Relations among Prior Probabilistic Decisions, the Path Probability Method, Optimal Entropy Inference and Statistical Mechanics

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In recent work on relational systems we have separated and outlined the minimal essential components of a physical theory.(1) Given only a delineation of the system variables and the experiential and/or formal relations thereon, but not the initial (prior) values thereof, this minimal composition assumes hierarchical form including the following levels:

- A) Order relations
  - 1) Deterministic kinematics
  - 2) Deterministic dynamics
- B) Initial order decision relations
- C) Choice of initial space-time relations
- D) Disorder relations
  - 1) Subsystemic probabilities
    - a) Probabilistic kinematics
    - b) Probabilistic dynamics
  - 2) Systemic probabilities
    - a) Probabilistic kinematics
    - b) Probabilistic dynamics
- E) Initial disorder decision relations
- F) Choice of initial probabilities
- G) Action
- H) Relation-to-the-real

It should be emphasized that this hierarchy is minimal, allowing order-disorder relations, and can obviously be extended. For example, one can admit additional decision (variational) processes for generating both the deterministic and probabilistic dynamics.

In this note we point out the relationships among prior probabilistic decision relations (maximum systemic probability principle, including the maximum entropy principle) and their application as variational principles for generating a probabilistic dynamics (path probability method and the method of optimal entropy inference) as well as how they generate statistical mechanics.

Within the disorder level there are included subsystemic probabilities, for which the kinematics corresponds to ordinary probability theory for classical systems, and a systemic probability associated with a system as a whole. The probabilistic dynamics is obtained either by formally imposing the deterministic dynamics on the probabilistic kinematics[1)~3)] or, if the deterministic dynamics is not given, by an additional decision (usually variational) process.[4)~6)] The resulting evolutionary equations can be written in solution operator form as

$$P_t = K_{t;to} P_{to}, \tag{1}$$

Where P_t and P_{to} are the time-evolved and prior probabilistic functions respectively and $K_{t;to}$ is the solution operator. Hence, within the relational hierarchy up through the probabilistic mechanics, one can formally generate a probabilistic function at time t if one is given the same function at some prior time t_o. To obtain this latter, a prior probabilistic decision relation is required. It is as follows: *choose those subsystemic probabilities which maximize the systemic probability subject to all given relational constraints*. Henceforth we will simply refer to this as the "decision process" herein.

It is easily recognized that the application of this "decision process" in classical systems generates what we know as equilibrium statistical mechanics given a particular form of the systemic probability (See e.g. reference 7)). If the logarithm of the systemic probability is employed along with Stirling's approximation, this "decision process" reduces to the form known as maximum entropy inference[8),9)] and again generates equilibrium statistical mechanics.

Moreover, if one is given a probabilistic kinematics but no probabilistic dynamics, this latter can be generated by a further application of this same "decision process". For Markoffian systems, this is exemplified by the now well known path probability method.[4),5)] Also to generate a probabilistic dynamics we subsequently developed, again for Markoffian systems, a specialization of the "decision process" employing the systemic probability in the particular logarithmic form given by the Shannon information-theoretic entropy.[10)] This method we have referred to as optimal entropy method can be seen to stand for specific constraints. Finally, we note that these methods subsume time-dependent statistical mechanics.[4)~6),9)]

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